THE NEW MATH SAT GAME PLAN

The Strategic Way to Score Higher

Philip Keller

The New Math SAT Game Plan

www.satgameplan.com

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Please note:

This is not a math textbook. And it's not one of those phone-book-sized review books, either.

I actually expect you to read this book from cover to cover.

If you do, and if you follow the instructions, your SAT math score will go up.

Now start reading.

"*Do you see how I got that problem right? It's all you-- I could hear your voice in my head: 'You just have to go a little slower, look a little deeper.' I actually enjoyed doing that problem.*"

-- Patrick C. (former student)

About the Author

Teaching SAT math has been Philip Keller's "other job" since 1985. While studying mechanical engineering at Princeton University, he taught SAT classes for a company called Pre-Test Review, a pioneer in SAT preparation. After graduating, he became a high school math and science teacher, teaching mostly physics but also chemistry, calculus and geometry. All through his years of teaching, he has continued to work with students to prepare them for the SAT.

He has also worked as a free-lance writer for the ACT, writing math and science questions, so he has seen the standardized testing world from inside and out. Currently, he teaches physics at Holmdel High School, in Holmdel, New Jersey. He also teaches the math classes for *Keller and White*, a small SAT and PSAT preparation program.

He lives in Shrewsbury, New Jersey, with his wife Daphne, his children Reuben and Jane, and his dogs Ranger, Stella and Pippin.

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INTRODUCTION

"Hey, Mr. Keller. You sound like an infomercial"

I know a secret about the SAT. Listen carefully.

There are many things in life that require patience and hard work. Many skills require years and years to learn. Many talented people find that they have to practice for a long time before they see improvement in their performance level, whether you are talking about playing golf, running a marathon, or playing the violin. No pain, no gain. You get out what you put in. Blah, blah, blah. OK, here comes the secret:

The math SAT is NOT ONE OF THOSE THINGS!

You will be surprised to see how easy it is to raise your score. But how can that be so? If there were an easy way, wouldn't everybody have heard about it by now? In this internet age of instant communication, how can there be a set of secret tricks that only a tiny percentage of test-takers ever find out about? I honestly have no idea! I think that kids should already know this stuff. But they don't. Time after time, I introduce my methods to students and I am met with astonishment, followed by relief, as one after another, they discover the easier way.

If you are like the vast majority of the students I have worked with, you have been attacking this test with the wrong strategy, using the wrong techniques, and badly mismanaging your time. These things are not hard to fix. And when you fix them, your score jumps up.

Then, you can review a small collection of math concepts and tricks that show up frequently on the SAT. After all, if even a half-dozen show up on your exam, and you have learned how to deal with them, then your score will take another upward jump.

And finally, you can take practice tests, where you apply your newfound knowledge and your new strategies and techniques. This way, you can monitor your progress as you practice what you have learned from this book. And as a result of all that practice, your score takes another jump.

I <u>know</u> these methods work. There are certain seasons of the year, shortly after the most recent SAT scores have been released, when I get the phone calls. Or students stop me in the hallway of the high school where I teach to give me their good news:

"Hey, thanks a lot. I went up 110 points in math."
"Hey Mr. Keller, you were right. I got a 680".
"Mr. Keller! I got an 800!"
"I went from a 450 to a 580. Can I take your course again?"

What most of these students are happiest about is how QUICKLY we accomplished this. The SAT course I teach meets for six sessions. It is unusual for me to work one-on-one with a student for more than 4 or 5 hours. Sometimes two or three hours is all it takes. In one extreme example, well, I'll quote the message that was left on my answering machine:

"Mr. Keller, I just want to thank you for taking the time to talk to my daughter. I know you only spoke to her for twenty minutes, but she did everything you said and her score went up 130 points."

I promise that's a true story. It is, of course, an extreme case, involving a student who was doing EVERYTHING wrong, but in a way that could quickly be adjusted. Like most students, she approached the SAT the way she learned to take tests in school. But when it comes to the SAT, the traditional school methods don't work for most students. Fortunately, there are non-traditional methods that DO work, methods that can be learned easily and quickly. It's not magic. It's not rocket science. It's not the miracle cure that you have seen on television. It's just that there is a better way to take this test, and when you learn it, you improve your score.

My first job is to convince you that it is possible to achieve a big increase quickly (but NOT effortlessly). I want you to have higher expectations. That's why sometimes I end up sounding like an infomercial. That's what I was told by one of my students on the first day of his SAT class. This student was actually kind of obnoxious. He wasn't too thrilled to be taking an SAT course and he was not the kind of student who always does what the teacher says. You could call him a non-conformist[*]. And he is precisely the kind of student I can help the most! After his SAT, he was just another one of those kids who thank me in the hall.

This just in: It happened AGAIN! I just got another phone call thanking me for the advice I gave over the phone. The student's score went up 190 points. I've never even met this kid. I just told her what to do over the phone. It really worked and it set my new record for largest, fastest improvement ever. This time, even I find it hard to believe. It's kind of spooky.

[*] You'll be hearing more about these non-conformists later. They tend to do well on the SAT, better in fact than their grades in school might lead you to expect. Meanwhile, hard-working, "good" students frequently "under score". It's important to learn why this happens and how to turn it to your advantage.

How to use this book: A guide for STUDENTS

(If you are a teacher, your guide is in the back of the book)

I have written this book to give my students a competitive advantage over the rest of the test-taking universe. I am going to help you raise your math score.

What you will have to do:

1. Well, obviously you have to read each section.

2. You have to work through the samples in the section and I mean actually work them out. You should be reading this book with a pencil and a calculator by your side.

3. After each section, there is a short practice set of problems for you to apply what that section teaches. The answers and explanations are on the pages after each problem set.

PLEASE NOTE: The practice problems are, in general, more toward the difficult side. Do NOT worry if you need to peek at the answers. It's good to practice with harder questions. But it's also important to practice with the right level problem. And that is why…

4. You have to take practice tests. At different points throughout the book, I'll tell you that it is time to go do a practice test. Ideally, you should do three or four actual practice tests before taking the real SAT. And in this case, more is better. Six would be good.

Wait a minute! This is a pretty skinny book. I don't see practice tests anywhere. How am I supposed to practice?

Yes, it's true. There are no full-length practice tests in this book. It's not because I couldn't make them up -- I write tests all the time. It's because if I did, they would not be the real thing. And there is no point practicing with fake tests. Think about it: if you were playing in the Superbowl, and you needed to scout your opponents, wouldn't you want films of their actual games? Or would you settle for a bunch of players dressed up to look like your opponent? I think you'd want the real thing. Same is true here…you need actual SATs. Most of the books in the SAT section of the bookstore are filled with fake SATs written by recent college graduates, typeset to look very much like the real thing. (I know because I was once one of those recent graduates. At the time, I never imagined that I would one day write actual questions for the ACT.) But you have to remember that the books are not real, and the tests in them are misleading. They often include material that ETS never uses, they are not at the right level of difficulty and they take longer than they should. Don't use them. ONLY PRACTICE WITH ACTUAL SATS! There are several ways for you to get them.

1. You can probably get one sample exam in a booklet at your school's guidance office.

2. You can ask older brothers and sisters or older friends if they sent away for their exam (It's called the 'Question and Answer Service' and it's available several times a year).

3. You can buy a book, put out by the College Board, called The Official SAT Study Guide. It costs about $15. If I were you, I'd buy it. We use it in the class I teach and I tell anybody I tutor to get it right away. Ignore the entire first half of the book, which is filled with useless information, bad advice and half-truths. The practice tests begin somewhere in the middle.
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How to Take a Practice Test

As you go through this book, you will see that I frequently compare the SAT to an athletic event. Partly, this is so I can use all the cool clichés that the coaches use. Here's one you may have heard once or twice: *You have to practice like you want to play.*

In this case, what that means is that when you take a practice test, you need to:

1. Work in a quiet room.

2. Work early in the day.

3. Review the Game Plan for your target score (see Lesson #1: It's About Time)

4. Get in the habit of writing on the test. Don't use scrap paper.

5. Use a timer and WORK FOR THE FULL TIME. This is the one rule that kids break all the time. When you are doing a practice test, you will be eager to get it done with as soon as you can. But as you will see, learning to manage your test time is a big part of improving, so it's critical that you practice the right way.

6. Take this seriously! Fight for every point you can! Pretend it's REAL!

Follow these directions EVERY time you take a practice test. If you can't do it right, wait until you have a better time to do the test. There are only a limited number of actual tests that have been released so you don't want to waste one by giving a half-hearted effort.

But there is a much more important reason for taking practice tests seriously: you will score higher. And that will make you happier. Even though it is just practice, when your score goes up, you will be excited to see the improvement and motivated to keep working (like losing weight on a diet). But if you don't take the practice test seriously and your score doesn't go up, you will have trouble maintaining your effort. After all, as we know, it's ok to hate the SAT, and to resent having to spend time working on your score. So here is a riddle. Maybe it will cheer you up.

Q*: *What's worse than spending time to improve your SAT score?

A*: *Spending the time and NOT improving your score.

But that doesn't have to happen. You CAN improve your score. The strategies that I'm going to teach you are not complicated. You will read them and think that they are obvious, common-sense ideas that anyone could use, if only someone would explain them. And that's exactly why I've written this book.

"Hey Mr. Keller, you want to hear something funny? I took [a nationally known SAT course] before I took your course. My score didn't change at all. Then I took your class and went up 120 points. I think that's funny."

- Student in line behind me at the cafeteria

Memo: About Calculators

Which one should you use? There are several ways to go here…

BASIC: You could just use a scientific calculator. Make sure that you know how to do fractions, decimals, exponents and roots.

ADVANCED: Use a graphing calculator. There may be one or two questions where it would be helpful to graph a line or graph a parabola. It is NOT essential, but having a graphing calculator would give you a slight edge.

MONSTER: Use a TI-89. This calculator is the most advanced one that is allowed on the SAT (and it definitely is allowed). But it is also intimidating to learn. Students have told me that they can't even figure out what to do after you turn it on. (You press the "Home" button.) And they complain that you tell it to divide and it doesn't. (Press the green diamond and then "Enter".) So why use it? I'll give you one really good reason: IT DOES ALGEBRA!

In other words, suppose you have an equation to solve. Let's say: $\frac{(3x-7)}{x}=7$

Instead of solving this by hand, you could type F2 and then "Enter". This would open the equation solving function. Then, you type the equation followed by ",x)". So your screen looks like this:

Solve((3x-7)/x=7,x)

And then you hit "Enter" and watch the calculator spit out the right answer: **x = -7/4** !

For me, that one feature makes it worth having this calculator. But many of my students worry about giving up their familiar calculator. The good news is that you don't have to! You can bring TWO calculators to the SAT: the one you are used to and the TI89. If the TI89 helps you to solve just one or two questions, that's 20 more points added to your math score.

Still, since many students choose to go with the simpler calculator, for most of this book I am only going to assume that you have a basic scientific calculator. But I will occasionally offer alternatives only available on a graphing calculator or TI89.

PART I: "I say, you're going about it all wrong"

- Famous Cartoon Chicken, Foghorn Leghorn

How the SAT is biased against "Good" Students
How eleven years of traditional math instruction have hurt your SAT score
How to unlearn what school has taught you about doing math and taking tests

The first thing I want you to remember is that this is not school. Although you take the SAT when you are in high school, it is NOT a high school test. It contains only a little bit of high school math, much of which can be avoided. It isn't structured like a school test. It isn't graded like a school test. So say it with me: "THIS IS NOT SCHOOL!" You do NOT have to do things the way you were taught. You don't get partial credit for doing things by the book. All that matters is whether or not you grid in the right answer.

Then, you also need to understand that the SAT is biased, but not the way that you might think: **The SAT is biased against the "good student" who does things in the way they are taught in school.** So who does this bias favor? You may not like this but here goes: it's the kid who looks to cut corners. He's maybe even a little bit lazy. But in a resourceful kind of way. He doesn't always do what he's told, and because of this he is penalized in school with grades that are lower than the "good student". But on the SAT, this student is rewarded. Let's call him the non-conformist. **The SAT is biased in favor of the non-conformist.** Is it you? If it is, then you have no doubt experienced some school-related grief because of your failure to conform. But the SAT is going to give you a break, <u>once you learn how to take advantage of it!</u> And what if it isn't you? Then you need to understand the situation and change your approach (just on the SAT!) so that you benefit from this bias instead of being hurt by it.

Do you need convincing? Take a look at this problem:

Given: $5x + 2y = 8$
$3x - y = 5$

Find: $2x + 3y$

It certainly looks like a typical high school algebra problem, and you might think that that's why it is on the SAT. But you'd be wrong.

Use the rest of the space on this page to solve the problem BEFORE YOU LOOK AT MY SOLUTION. (I told you that you would have to work, too, remember? So do it.)

On the next page, I'm going to solve this problem for you, twice. First I'll do it the way you are taught in school. Then I'll do it the nonconformist's way. You decide which one you like better.

Given: $5x + 2y = 8$
$3x - y = 5$

Find: $2x + 3y$

Solution Method #1…the school way:

This is a pair of 2 equations and 2 unknowns.
I will use the method of simultaneous equations.

First I double the second equation.
That makes the number in front of the y's the same.

Then I add the equations.

Then solve for x.

Then plug x back into one of the two equations.

Then solve for y.

Now plug x and y into the expression we were asked to solve.

$$5x + 2y = 8$$
$$6x - 2y = 10$$
$$11x = 18$$
$$x = \frac{18}{11}$$

$$3 \times \frac{18}{11} - y = 5$$
$$\frac{54}{11} - y = \frac{55}{11}$$
$$y = -\frac{1}{11}$$

$$2 \times \frac{18}{11} + 3 \times (-\frac{1}{11}) =$$
$$\frac{36}{11} - \frac{3}{11} = \frac{33}{11} = 3$$

Hello? Hello? Are you still there? Did you fall asleep in the middle of that? Or even worse, are you now completely freaked out with anxiety because there's no way that you could do that problem in less than 20 minutes, if at all? Relax. Take deep cleansing breaths. Now look at method #2.

Given: $5x + 2y = 8$
$3x - y = 5$

Find: $2x + 3y$

Solution Method #2…the non-conformist way:

Hmm. There must be a way to get from the given information to the answer.

Let's see what happens if I add the equations…

$$\begin{array}{r} 5x + 2y = 8 \\ \underline{3x - y = 5} \\ 8x + y = 13 \end{array}$$

No, that's not it.

Ok, what if I subtract the equations?

$$\begin{array}{r} 5x + 2y = 8 \\ \underline{3x - y = 5} \\ 2x + 3y = 3 \end{array}$$

Hey look! That's it! I'm done.

OK, let's look at what just happened there. The poor kid who uses high school algebra needs a whole page of ugly math to solve this question and probably two or three minutes. But the non-conformist who plays around, hoping for a shortcut, gets his wish. And takes less than a minute. And it's lucky for him, because he could not have solved it the hard way -- he wasn't paying attention in class that day. He didn't do well on his math quiz that week, but he got it right on the SAT.

Now, some of you are thinking: "Wait a minute. That's cheap. The non-conformist was just lucky that adding or subtracting the equations just happened to land you on what you need. They could have written the problem with different numbers so that you HAVE to do it the hard way."

To which I respond: yes they COULD have written it that way, but they DIDN'T. From their point of view, the goal of the question was to see if you could find the easy connection. But if you want to torment yourself and waste time doing it the long, formal, in-school way, they will let you. But the non-conformist will finish faster than you and move on to the next question. Do you see what I mean? This test is biased. It's not fair.

What it comes down to is this: if you have been a non-conforming student your whole life, if you sleep in class, if you've occasionally been known to copy some one else's homework, or to trust your creativity to get you through tests, well then the SAT is written for you (and I think you are going to LOVE this book).

On the other hand, what if you are a "good" student? You do all your work, you take notes, you pay attention in class, you're a solid student, but the SAT is not designed for you. In fact, you know some of those non-conformists with lower grades than you but higher SAT scores. And it's making you mad. Talk about unfair. Let me reassure you, though. You have been a good student, good at learning what you have been taught. Your hard work has usually been rewarded. So now, I'm going to teach you something new – how to be creative and sneaky and how to cut corners. You can learn this way too. You might enjoy it, and it will raise your score. And then, back in school, you can continue to be the responsible, hard-working student that you have been all along. Only now you will be a hard working kid who also knows a few tricks.

Let's check your understanding so far:

Q: Why don't we want to use school methods on the SAT?

A: Because the SAT is NOT SCHOOL.

Try to keep that in mind as you continue working through this book.

Please turn the page and begin Lesson #1.

"I had a 460 on the first practice test, two weeks ago. Now I'm up to a 560. How sweet is that?"

- Sarah G.

Lesson #1: It's About <u>Time</u> !

If I could only teach you one thing about taking the SAT, this is what it would be:

SLOW DOWN!!! Stop trying to finish the sections!

Unless you have already earned above a 700 in previous practice tests, finishing the section is a mistake. I usually spend the first 30 minutes of every SAT course I teach explaining why this is true. I know it is hard to believe, for two reasons.

Reason #1: ETS tells you not to spend too much time on any problem. It's written in many of their pamphlets and is even part of the instructions the proctors read to you on the day of the test. ("Skip it and move on. You can return to the problem later if time remains." – sound familiar?)

Reason # 2: It goes against all of your school experience. Think about it. In school, you ALWAYS try to finish tests. If the bell rings before you get to the last page, you've had a bad day ("Oh, man! That test was impossible. I couldn't even finish!"). After all, those problems on the last page are usually the hardest ones and they are usually worth the most points. So in school, you need to make certain that you spend some time on them and at least earn some partial credit.

But, as I may have mentioned, THIS IS NOT SCHOOL! The SAT has a different set of rules:

<u>The hard problems and the easy problems are worth the same number of points!!!</u>

That may seem unfair. (Everything about the SAT seems unfair.) But we can adjust your strategy to take this into account.

<u>There is no partial credit for wrong answers. In fact, there is a PENALTY.</u>

In school, wrong answers are usually scored the same as blanks. But on the SAT, there is a penalty for getting things wrong. So your courage is NOT rewarded. In school, the grading system most teachers use encourages you to at least TRY every problem. By giving partial credit, we teachers send the message that ANYTHING is better than nothing. The SAT does just the opposite, so your strategy has to take this into account also.

<u>You can leave out a LOT of questions and still get a really good score.</u>

To give you a rough idea…

… You can omit 20%, get another 10% wrong and still be in the 600 neighborhood.
… You can omit 30%, get another 20% wrong and still be in the 500 neighborhood.

But why would you omit so many questions? Because they are HARD! And because you will lose points if you get them wrong. Your strategy has to take this into account, too.

So you see, the SAT is a different game, played by a different set of rules. And the first rule is to GO SLOW! There are problems on every SAT that you've been getting wrong simply because you were rushing. You rush because the little voice in your head says, "Move along, keep going or

you'll never finish." But why would you want to finish? When you slow down, you will start getting those problems right, unlike those hard problems at the end, which you weren't getting right anyway. So why rush?

Still not convinced? Well, consider the following analogy…

The Basketball Shooting Contest

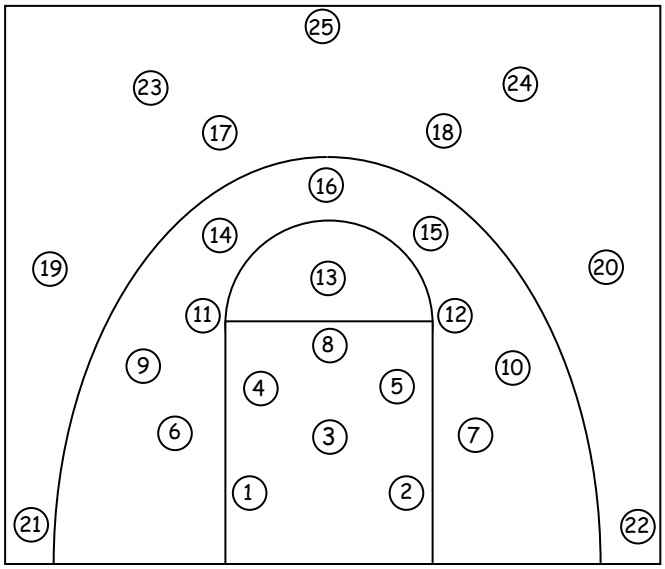
Suppose you are going to be in a basketball-shooting contest, not for something trivial like college admission, but for something you really care about: money!

The rules of the contest are:

1. You have 1 minute to shoot up to 25 times.

2. You may shoot only once from each of the numbered spots whether you make the shot or not.

3. You get $100 for every basket you make.



OK. You want to win the most money that you can. What's your strategy?

"Uh, I think I'll shoot as well and as fast as I can and hope to make a lot shots."

Well that's a good idea. In what order will you shoot them?

"I don't know. I might want to get the hard shots out of the way first."

Now that's NOT a good idea. You'd feel silly if time ran out before you shot the easiest ones.

"Oh, yeah. OK, easy shots first, hard ones later. Good thinking, coach."

Why are you shooting quickly?

"*To shoot all 25 possible shots. Can't make 'em if you don't shoot 'em. Duh"*

OK, good point. But wait a minute. Here comes a twist…

The Crucial Twist

Now, let's add a crucial twist: Suppose we change the rules so that while you still get $100 for every basket you make, you now LOSE $25 every time you miss. Does that change anything?

Think for a moment. Do you still want to shoot all the shots? Are you still going to hurry? NO and NO! What should you do instead? Now, your best bet is to shoot the shots that you have a pretty good chance of making, and that's all. You should NOT shoot any shot that is beyond your range. It wastes your time and costs you money. You are better off taking more time with the shots that you do have a good chance to make.

That's the general idea, but what if you need me to be more specific? What if you are thinking, "Tell me exactly which shots to attempt and which to skip."? I can't -- there is no single strategy that would work for everybody. Each player has to develop his or her own ideal game plan. To maximize your potential score now requires PLANNING.

First, you have to practice shooting. It will make you a better shooter, but it will also help you to learn what shots you are good at and what shots you are not.

Then, you have to decide in advance what shots you are going to take. In other words, what shots are worth risking $25 to attempt?

Finally, on the day of the contest, you attempt only those shots, and you take your time. You do NOT care that time will run out before you shoot the hard shots. Why not? Because:

1. By shooting slowly, you are making a higher percentage of the shots that are in your range.
2. The shots you didn't attempt are shots you probably would have lost money on anyway.

Even if time were not a factor, only the best shooters should attempt those last few shots. Most people would just be throwing money away. *"But sometimes I make that shot! I really think I can do it."* Yes, sometimes you do make that hard shot. But is it a good risk? It costs you money and time. You need to make a game plan, and follow it if you want to win at this game.

"But in basketball, I know how hard every shot is before I shoot it. On the SAT, how am I supposed to know how difficult the question is if I don't try to answer it?"

That is a really important question. The answer is…(drum roll, please**)… the questions are arranged in order of difficulty**. In every section, and every subsection, the early questions are easy, the later questions are harder and the last questions are just plain nasty. Knowing this, you can plan your strategy in advance, just like the basket shooter.

HOW TO MAP OUT YOUR GAME PLAN

Step 1: Set your score goal
Step 2: Plan which questions to answer and which to IGNORE

Determining Your Goal

Your strategy is based on the score you are aiming for. You should aim to improve your math score by around **100 points** from your most recent SAT or PSAT. Students who start with a lower score can expect bigger improvements. They have more room to improve and it's easier when you start lower. On the other hand, students who are already scoring high (let's say in the 600's) will find it harder to go up by as many points. Yes, the curve gets steeper at the top, but also, these students have less room to improve. Still, these techniques can raise their scores. One of my favorite students came to my SAT course with a 740 in math. He was really taking the course for the verbal help. But he paid attention to all of the math tricks and was happy to report to me that they helped him to an 800. He's at MIT now.

If you have never taken the SAT or PSAT, start by aiming for a 550 or 600. But then, you MUST do a practice test to see if you are in the right range.

Your "Raw Score" and the penalty for wrong answers

Your raw score is the number of questions you answer correctly, minus the penalties for all the ones you got wrong. Here's how it works: on every question, one of three things happens…

1. If you get it RIGHT, you earn 1 raw point.
2. If you leave it BLANK, you don't earn any raw points, but you don't lose any either.
3. If you get it WRONG, you lose a quarter of a point on a multiple choice question. You don't lose any points for wrong grid-ins (but as you will see later, that does NOT mean you should answer them all). Another way to think of it is that in the 5-choice section, every four wrong answers cancel one of your right answers.

There is also a hidden penalty. Can you guess what it is? In addition to costing you points, getting things wrong is a WASTE OF TIME! (Think back to the basketball analogy.) That's why you want to learn and follow your game plan.

What's in a Game Plan?

The Game Plans show you what to answer and what to omit in each of the sections and sub-sections of the math SAT. There are three types of sections that you can tell apart instantly by the number of questions in each:

20 Questions: you get 25 minutes to answer 20 multiple questions of steadily increasing difficulty.

18 Questions: you get 25 minutes to answer 8 multiple choice questions followed by 10 "grid-in" questions. And each set follows the easy-medium-hard progression. This means that #7 and # 8 are substantially harder than the next few questions that follow them.

16-Questions: you get 20 minutes to answer 16 multiple choice questions and again they get harder as you go.

More Bad News: The Experimental Sections

In case you are not already ticked off about having to take the SAT, here's something really irritating that you may have not heard about: the "experimental" sections. Of the more than three hours you spend testing, twenty-five minutes are spent taking a section that won't count toward your score. You might get an extra 25-minute math section on your SAT. And if you don't, you will get an extra 25-minute verbal section instead.*

The extra section does not count toward your score.
There is no way to tell which section is the "extra" one.
You have to do ALL of the sections as best you can.
ETS is using 25 minutes of your time to gather information for future tests.

THERE IS NOTHING YOU CAN DO ABOUT THIS!!!

Yes, I know, it's not fair…unfairness is everywhere. If it makes you feel better to know it, they use the information from the experimental section to figure out whether a problem is hard or easy. And then they put them in order for you. So this is an injustice that you benefit from in a small way. (But it still stinks.)

IMPORTANT NOTE:

The score goal you select now is only your FIRST STEP!!!

I expect my students to reach their first goal in a matter of weeks. And when you do, when you earn that score on a timed practice test, you should then look up the Game Plan for scoring 50 points higher, and start working toward that score. My most successful students have improved their math scores by 200 points when they were through. It could happen to you.

The Game Plans begin on the next page. Find the one that matches your goal and study it, but don't worry about memorizing it. You can even leave it open in front of you when you are taking your practice tests. By the time you have done a few practice tests, you will know your plan by heart. You can even bring your plan with you on the morning of the SAT and review it right up to the last minute. (Of course, you have to put it away when the test starts.)

The game plans on the next pages are also available on our website: www.satgameplan.com – you go to the website and then choose the "Interactive Math Strategy Guide". Then, click on the score you are aiming for and follow the instructions on the chart.

* That's why when you look at the exams in "The Official SAT Study Guide", the sections are numbered 1 – 10, but one of the numbered sections is missing. The missing section was the experimental section, which they do not include when they release the test to the public.

PRACTICE TEST ALERT: Once you have learned your Game Plan, it would be a good time to take a TIMED practice test (from *The Official SAT Study Guide*) to see how much easier it is when you use time properly.

Remove this page and study both sides before you take your SAT!

Target Score: 500 Keller's Math SAT Game Plan

Work your way SLOWLY through your "Answer Zone". Be PATIENT with yourself and give yourself time to READ, THINK and PLAY. Don't skip easier questions just to end up wasting time on harder ones!

Section-by-section Breakdown:

20 Question Section

Answer Zone												Skip Zone							
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**

18 Question Section

Answer Zone					Skip Zone			Answer Zone						Skip Zone			
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**

16 Question Section

Answer Zone										Skip Zone					
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**

What is the best score I can get with this strategy?

If you answer all of the "Answer Zone" correctly, you will score around a 550. You can miss 6 or 7 and still get a 500!

Should I EVER attempt a question in the "Skip Zone?

Yes, but only if you happen to recognize a specific problem type where you have learned the trick. For example, the "Back Door play" (see Lesson #3) shows up often in the "Skip Zone", so that gives you a chance to steal some points.

Can I EVER skip a question inside the* "Answer Zone"*?

Hmmm. Well. Yes, but only if it's late in the zone and it's on a topic that you are especially weak in. For example, some students will skip late geometry questions. But the basic rule is: once you spend time on it, ANSWER IT!

What if a "Skip Zone" question looks easy?

Unless it matches a method you have learned in this book, I'd be VERY careful. The SAT is very good at making things look easier than they are. It's in the "Skip Zone" for a reason.

What if an "Answer Zone" question seems to hard?

You are probably over-thinking, or even more likely, misreading the question. Go back and take another look, SLOWLY! It's going to be easier than it looks. It's in the "Answer Zone" for a reason.

ALSO STUDY THE REMINDERS ON THE OTHER SIDE OF THIS SHEET!!!

Things to remember as you are about to begin:

1. Go SLOOOOWWWW. As slow as you can make yourself go. The questions you are rushing through are easier than the ones that you are rushing to.

2. PLAY!!! Draw pictures. Make lists. Take guesses. It's NOT school.

3. Avoid algebra! Go to their answers and ask, "Could THIS be it? Let's see…"

4. Avoid algebra! Use the Back Door play*. Make up your own numbers that fit.

5. It's going fine. It isn't any harder than the practice tests you have done. It just seems harder now that it counts. Take a deep breath, calm down, and keep playing.

6. USE YOUR CALCULATOR (Especially, TI89! F2, Enter…)

7. READ EVERY WORD! The words you skip are the reasons you get an easy question wrong.

8. Be slow and bold! Once you spend time on a question, go ahead and answer it. But don't be in a hurry to answer more questions than you should. FOLLOW YOUR GAME PLAN!

9. Take your breaks! You are entitled to one break after every hour. Use it. Get up. Wake up. Have a drink. Eat a snack. Get ready to start a new section.

10. Relax, and play your game. Don't worry if you think you've gotten one wrong. You are going to make mistakes. That's OK. Just stay in your plan, and keep doing your best.

* Covered in Lesson #3

Target Score: 550 Keller's Math SAT Game Plan

Work your way SLOWLY through your "Answer Zone". Be PATIENT with yourself and give yourself time to READ, THINK and PLAY. Don't skip easier questions just to end up wasting time on harder ones!

Section-by-section Breakdown:

20 Question Section

Answer Zone														Skip Zone					
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**

18 Question Section

Answer Zone						Skip		Answer Zone						Skip Zone			
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**

16 Question Section

Answer Zone											Skip Zone				
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**

What is the best score I can get with this strategy?

If you answer all of the "Answer Zone" correctly, you will score around a 590. You can miss 4 or 5 and still get a 550!

Should I EVER attempt a question in the "Skip Zone?

Yes, but only if you happen to recognize a specific problem type where you have learned the trick. For example, the "Back Door play" (see Lesson #3) shows up often in the "Skip Zone", so that gives you a chance to steal some points.

Can I EVER skip a question inside the* "Answer Zone"*?

Hmmm. Well. Yes, but only if it's late in the zone and it's on a topic that you are especially weak in. For example, some students will skip late geometry questions. But the basic rule is: once you spend time on it, ANSWER IT!

What if a "Skip Zone" question looks easy?

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4. Avoid algebra! Use the Back Door play*. Make up your own numbers that fit.

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7. READ EVERY WORD! The words you skip are the reasons you get an easy question wrong.

8. Be slow and bold! Once you spend time on a question, go ahead and answer it. But don't be in a hurry to answer more questions than you should. FOLLOW YOUR GAME PLAN!

9. Take your breaks! You are entitled to one break after every hour. Use it. Get up. Wake up. Have a drink. Eat a snack. Get ready to start a new section.

10. Relax, and play your game. Don't worry if you think you've gotten one wrong. You are going to make mistakes. That's OK. Just stay in your plan, and keep doing your best.

* Covered in Lesson #3

Target Score: 600 Keller's Math SAT Game Plan

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Section-by-section Breakdown:

20 Question Section

Answer Zone																Skip Zone			
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**

18 Question Section

Answer Zone						Skip		Answer Zone							Skip Zone		
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**

16 Question Section

Answer Zone													Skip Zone		
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**

What is the best score I can get with this strategy?

If you answer all of the "Answer Zone" correctly, you will score around a 640. You can miss 3 or 4 and still get a 600!

Should I EVER attempt a question in the "Skip Zone?

Yes, but only if you happen to recognize a specific problem type where you have learned the trick. For example, the "Back Door play" (see Lesson #3) shows up often in the "Skip Zone", so that gives you a chance to steal some points.

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7. READ EVERY WORD! The words you skip are the reasons you get an easy question wrong.

8. Be slow and bold! Once you spend time on a question, go ahead and answer it. But don't be in a hurry to answer more questions than you should. FOLLOW YOUR GAME PLAN!

9. Take your breaks! You are entitled to one break after every hour. Use it. Get up. Wake up. Have a drink. Eat a snack. Get ready to start a new section.

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* Covered in Lesson #3

Target Score: 650 Keller's Math SAT Game Plan

PLEASE NOTE: A HIGHER SCORE GOAL REQUIRES YOUR GAME PLAN TO BE MORE SUBTLE…

There are no questions that are automatically too hard for you to try. However, you need to use time wisely. Start by moving SLOWLY through the "Answer Zone". Your goal is to sweep those questions. Then, only if you still have time, pick and choose one or two questions from the "Scan Zone." In the end, you should attempt not quite half of the "Scan Zone". But how do you choose? Look for:

1. **Problems that you can do by trial and error**
2. **Problems that you can do by the Back Door Trick**
3. **Problems that don't look like high school math—the stranger the better!**

In general, you'll find that you can trust your own judgment. And that means if you don't like ANY of the "Scan Zone" questions in a given section, feel free to skip them all. Your first job is to sweep the "Answer Zone".

Section-by-section Breakdown:

20 Question Section

Answer Zone																Scan Zone			
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**

18 Question Section

Answer Zone						Scan		Answer Zone							Scan Zone		
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**

16 Question Section

Answer Zone													Skip Zone		
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**

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* Covered in Lesson #3

Target Score: 700 Keller's Math SAT Game Plan

Plan on attempting nearly all of the questions. Plan on ignoring the two or three hardest questions.

PLEASE NOTE: A HIGHER SCORE GOAL REQUIRES YOUR GAME PLAN TO BE MORE SUBTLE…

There are no questions that are automatically too hard for you to try. However, you need to use time wisely. Start by moving SLOWLY through the "Answer Zone". Your goal is to sweep those questions. Then, only if you still have time, pick and choose one or two questions from the "Scan Zone." In the end, you should attempt two thirds of the "Scan Zone". But how do you choose? Look for:

1. **Problems that you can do by trial and error**
2. **Problems that you can do by the Back Door Trick**
3. **Problems that don't look like high school math—the stranger the better!**

In general, you'll find that you can trust your own judgment. And that means if you don't like ANY of the "Scan Zone" questions in a given section, feel free to skip them all. Your first job is to sweep the "Answer Zone".

Section-by-section Breakdown:

20 Question Section

Answer Zone																Scan Zone			
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**

18 Question Section

Answer Zone						Scan		Answer Zone							Scan Zone		
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**

16 Question Section

Answer Zone													Skip Zone		
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**

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* Covered in Lesson #3

Lesson #2: No Algebra, Please.

We LOVE trial and error. And it's OK to be wrong.

There is something strange about how we teach math in high school. There is very little time for playful exploration, for making guesses, for daring to be wrong. Most students experience math class as a place where you are taught the proper methods for solving a specific set of problem types: This is how you complete the square. This is how you rationalize a denominator. This is how you graph a cosine function. Follow the procedure. Do it my way. Guessing has no place here. The only people who guess are the truly desperate.

Now maybe this is just the way it has to be (though I doubt it). But guessing is a legitimate mathematical technique. Professional mathematicians use it all the time, especially when they are just beginning to work on a problem. Now, you may not go on to become a professional mathematician, but you *are* going to take the SAT. Learning to guess and learning to play with a problem will give you an advantage.

So, here is the SECOND piece of advice I give to students (this is Lesson #2, for those of you who are keeping score at home).

Don't feel guilty about guessing. Don't feel guilty about using trial and error.

Whenever you are stuck, especially if you are lost in some algebraic mess, use trial and error. One at a time, with either their answer choices or your own guesses, keep asking yourself:

"Could THIS be the answer? What if it were? Let's see if it works…"

Then, play with it and see if it fits the question. If it does, great – you've got the answer. If not, try another guess.

I'm going to show you a bunch of examples of SAT-type problems that COULD be solved algebraically. But the solutions I will demonstrate do not use algebra. If that makes you happier, skip the next paragraph and go to the first sample. But if the thought of avoiding all that algebra makes you uncomfortable, because you PREFER doing algebra, the next paragraph is for you.

"But I like doing algebra…"

Don't feel embarrassed. Actually, you are not alone. Many students tell me that they are more comfortable using algebra to solve problems. And that's OK. But on the SAT, there are times when you will not be able to come up with the right equations to solve. Or you just might not be able to solve them. Please be open-minded and try my way. Then, at least you will have a new alternative. Besides, I predict that once you get the hang of it, you'll see that trial and error is an amazingly useful trick.

Example #1: First try this by using algebra— **If $(2x - 2)^2 = 64$, and $x>0$, find x.**

How did you do? Well, let's look at the answer choices:

a) 4 b) 5 c) 6 d) 7 e) 8

I am NOT going to show you the algebraic solution. Instead, I am going to ask you a question:

Could the answer to this question be 'a'? In other words, does $x = 4$?

You might think that you can't answer that question if you have not found the solution yet. But you CAN! Just take x=4 and plug it into the equation and see if it works!

$(2x - 2)^2 = 64$

Is $x = 4$?

Let's see…2 times 4 is 8. $8 - 2 = 6$. $6^2 = 36$, which is NOT 64. So 'a' is NOT the answer.

Now what? Oh, yeah…let's try the next answer choice.

Is $x = 5$?

Let's see…2 times 5 is 10, $10 - 2 = 8$, $8^2 = 64$…It works! I'm done!

So what have we learned here? It's that, when it comes to finding answers to algebraic equations,

Checking solutions is easier than finding solutions!!!

You would think that ETS would stop putting this kind of question on the test. If there is just a single equation for you to solve, the trial-and-error method makes it simple. In fact, there are certain graphing calculators (such as the TI-89) that can solve these equations for you. Still, ETS continues to use this problem type. But they do try to make it a little harder…Here's one with two equations and two unknowns:

$$2x + y = 12$$
$$x^2 + 2y^2 = 33$$

Find a possible value for x.

The school method involves solving the first equation for y, substituting the expression into the second equation, "foil"-ing, distributing, regrouping, factoring and then solving. Try it here…

Now tell the truth: did you really try the school method? Or did you just skip it? If you skipped it, then….GOOD JOB! I'm proud of you. It means that you are getting the idea that the school way wastes time. Let's try it another way…

$$2x + y = 12$$
$$x^2 + 2y^2 = 33$$

What if we look at the answer choices and play "Could THIS be it?"

a) 3 b) 4 c) 5 d) 6 e) 7

Is x=3? Let's see….$2 \cdot 3 + y = 12$, $6 + y = 12$ so $y = 6$.
But then $3^2 + 2 \cdot 6^2 = 81$, which is NOT 33. So x isn't 3.

Is x=4? Let's see….$2 \cdot 4 + y = 12$, $8 + y = 12$ so $y = 4$.
But then $4^2 + 2 \cdot 4^2 = 48$, which is NOT 33. So x isn't 4.

Is x=5? Let's see….$2 \cdot 5 + y = 12$, $10 + y = 12$ so $y = 2$.
But then $5^2 + 2 \cdot 2^2 = 33$, which is CORRECT!!!

Now, students sometimes object to this procedure, claiming that it takes too long. To which I have three responses:

1. What's your hurry? You are supposed to be taking your time!

2. After you practice the trial and error method for a while, you get quicker at it.

3. Did you get this right using school math? Or did you even try? By traditional methods, this is a pretty tough problem. ***Using a method that takes time but actually gets you the answer is better than not getting the answer at all, fast or slow!***

TRIAL AND ERROR WITH WORD PROBLEMS

The last examples showed what to do when the problem gives you equations that you have to solve. But they don't always give you the equations. Sometimes, they expect you to figure out the equation for yourself. I'm talking about word problems. Many of you dislike word problems so much that you skip them almost automatically. But you don't have to skip them, and you don't have to do algebra. Trial and error works here as well.

Let's see how this works…

The sum of four consecutive integers is 82. Find the greatest of the four integers.

If you have not been convinced yet, feel free to do this problem by algebra.
The non-conformist method is shown below. (And the non-conformist himself has already skipped to the solutions!)

The sum of four consecutive integers is 82. Find the greatest of the four integers.

a) 19 b) 20 c) 21 d) 22 e) 23 [I'm going straight to the easy way.]

Is it 'a'? In other words, is 19 the greatest of the four consecutive integers?
Then, the integers would have to be 16, 17, 18, 19. Let's see if it works.
16+17+18+19=70, which is NOT 82. It's too small. I think I'll try 'e' next.

Is it 'e'? In other words, is 23 the greatest of the four consecutive integers?
Then, the integers would have to be 20, 21, 22, 23. Let's see if it works.
20+21+22+23=86, which is NOT 82. It's a little too big. I think I'll try 'd' next.

Is it 'd'? In other words, is 22 the greatest of the four consecutive integers?
Then, the integers would have to be 19, 20, 21, 22. Let's see if it works.
19+20+21+22=82, which is CORRECT. And we're done. With no algebra!

In the examples that follow, I have chosen to make 'B' the correct answer. This is because, for the purpose of teaching you this method, it is important that you see at least one wrong answer before seeing the answer that turns out to be right. On the actual SAT, there is no pattern. Sometimes, the first answer you try works and sometimes you have to try them all. There is no pattern. None. In fact, paying attention to the pattern of your answers is one of those "*50 Habits of Insane People".* Please do not interpret my using 'B' so frequently as a recommendation to choose 'B' on the actual test. There is NO pattern. None. Now, more examples…

On a certain bridge, there is a toll of $3 for cars and $5 for trucks. From 50 vehicles, all either cars or trucks, a total of $210 in tolls was collected. How many of the vehicles were cars?

a) 15 b) 20 c)….whatever….

Is it 'a'? If there were 15 cars, then since 50 – 15 = 35, there were 35 trucks.
The 15 cars pay 15·3 = $45, the 35 trucks pay 35·5 = $175.
$45 + $175 = $220, which is not $210 so the answer is NOT 'a'.

Is it 'b'? If there were 20 cars, then since 50 – 20 = 30, there were 30 trucks.
The 20 cars pay 20·3 = $60, the 30 trucks pay 30·5 = $150.
$60 + $150 = $210, which is CORRECT! We are done. And no algebra, again!

* * * * * * * * * *

Here's a really obnoxious one[*]…

Mary is 15 years older than Tom. Nine years from now, her age will be three less than twice what his age is then. How old is Tom now?

a) 3 b) 9 c)…whatever…

Is it 'a'? If Tom is 3, then Mary is 3+15=18.
So in 9 years, Tom will be 3+9=12 and Mary will be 18+9=27.
But 27 is NOT 3 less than twice 12. So the answer is not 'a'.

Is it 'b'? If Tom is 9, then Mary is 9+15=24.
So in 9 years, Tom will be 9+9=18 and Mary will be 24+9=33.
And 33 IS 3 less than twice 18. So 'b" is CORRECT! We are done. And no algebra, again!

* * * * * * * * * *

Most SAT's include three or four problems that can be done by trial and error, which is reason enough to learn this method. But also, as you learn this method, you start to let go of your instinct to do things the way you do them in school. When that happens, your score rises, because (as I may have mentioned once or twice) THIS IS NOT SCHOOL.

[*] You probably hate this kind of problem. Here's why: it's STUPID and pointless. Algebra books usually include problems like these, ironically, to demonstrate how useful algebra really is. I think of this kind of problem as the mathematical equivalent of word-search puzzles – fun, if you like that sort of thing, but useless.

Here's a silly one:

At a family picnic, children and dogs are playing in the backyard. Aunt Lisa looks out in the yard and counts 18 heads and 50 legs. How many dogs are in the yard? (You may assume that the dogs have 4 legs each, and the kids have 2 legs each.)

a) 7 b) 8 c) 9 d) 10 e) 11

OK, this time, suppose we start with answer choice c:

Suppose there are 9 dogs. Then they have 9x4=36 legs. So we need 14 more legs to get to 50. That means 7 kids. But 9+7 does not add up to 18 heads. So it's not 'c'.

Now, we can save time if we notice that to get more heads, we need fewer dog legs. If you don't notice this, that's OK too. Just pick another answer and start playing. So let's try 'a':

Suppose there are 7 dogs. That would be 7x4=28 legs. Need 22 more to get to 50. That means 11 kids. And 11 + 7 = 18. Yes!

(You could also reason this out in the opposite order: start by making the heads come out right and then work on the legs. Either way works.)

Before you turn to the practice problems, let's look back at what have we learned:

1. Don't do algebra. Don't write equations. Don't solve equations.
2. Go to each answer choice and PLAY with it…see if it works.

This is assuming, of course, that there are answer choices to choose from. But what if you are working in the "Grid-in" section, where there are no answer choices? You can STILL use trial and error. I will show you how in Lesson #4. But for now, there are practice problems on the next page which can all be done by trial and error. In addition, when you move into Part II of this book, and you are working on those practice problems, be on the lookout for trial and error problems there as well.

Practice Set #1: Trial and Error

1. If three more than a number is the same as three times that number, then the number is:

a) 0 b) 1 c) 1.5 d) –3 e) –1/2

2. The first of four consecutive even integers is 32 less than the sum of the last three. What is the first of the four?

a) 10 b) 12 c) 14 d) 15 e) 16

3. A theater sells general admissions tickets and reserved seating tickets. A reserved seating ticket costs $4 more than a general admission ticket. When they sold 120 general admission seats and 80 reserved seats, they took in a total of $1920 in sales. How much did they charge for the general admissions ticket?

a) $5 b) $6 c) $7 d) $8 e) $9

4. Jack drives home from college every few weeks, usually averaging 50 miles per hour for the trip. Then, he discovered that if he increased his average speed to 60 miles per hour, he could shorten his trip by 30 minutes. How long is the trip?

a) 75 miles b) 125 miles c) 150 miles d) 175 miles e) 300 miles

5. When a number is divided by 6, the remainder is twice as big as the remainder you get when the same number is divided by 5. Which of the following could be the number?

a) 18 b) 22 c) 28 d) 45 e) 49

6. Marcia is three times as old as Barbara. But six years from now, she will only be twice as old as Barbara. How old is Barbara now?

a) 4 b) 6 c) 9 d) 12 e) 15

Set #1 Answer Key

For each one of these, the method is the same: pick an answer choice, see if it works. If not, pick another…
(For those of you who insist on using algebraic solutions, go ask your algebra teacher ☺)

1. We are going to add 3 to a number and see if we get the same result as when we triple the number instead, starting with answer 'a' and moving on…
0+3=3, 0x3=0…no
1+3=4, 1x3=3…no
1.5+3=4.5, 1.5 x 3 = 4.5…YES!
ANSWER: c

2. If 10 is the first of the four then the numbers are 10, 12, 14, and 16. The sum of the last three is 12+14+16 = 42. And, since 42 IS 32 more than the first number, answer 'a' is correct.
ANSWER: a

3. Could the answer be 'a'? If the general admission seats are $5, then the reserved seats are $4 dollars more which makes them $9. So the money they take in is $5 x 120 + $9 x 80 = $1320. But that's too low. So next I'll try 'c'.
If the general admission are $7 then the reserved are $11. $7 x 120 + $11 x 80 = $1720…still too low.
Now 'd': If the generals are $8 and the reserves are $12, then it's $8 x 120 + $12 x 80 = $1920. YES!
ANSWER: d

4. Let's try 'e' first: If the trip is 300 miles, then at 50 miles per hour it would take 300/50 = 6 hours. But at 60 miles per hour it would take 300/60 = 5 hours. That's a savings of a full hour or 60 minutes. So that's not the answer. In fact, it's twice as much savings as we are looking for! So a good next guess would be 'c'. (But if you don't realize this, it doesn't matter – you'll still get it eventually.) If the trip is 150 miles, then at 50 miles per hour, it would take 150/50=3 hours. And at 60 miles per hour, it would take 150/6 = 2.5 hours for a savings of a half hour, which is 30 minutes. **ANSWER: c**

5. First of all, if you are in need of a review of remainders, see page 69. Once you remember how to find remainders, this question is just a matter of checking each choice:
a) 18/6 = 3 r 0 and 18/5 = 3 r 5…but 0 is not twice as big as 5.
b) 22/6 = 3 r 4 and 22/5 = 4 r 2…4 IS twice as big as 2
ANSWER: b

6. Is Barbara 4? Then Marcia must be 12 (which is 3x as old). So 6 years from now, Barbara will be 10 and Marcia will be 18. But 18 is not twice as old as 10. So Barbara is not 4.
Is Barbara 6? Then Marcia would have to be 18 (which is 3x6). And 6 years from now, Barbara would be 12 and Marcia would be 24. And since 24 IS twice as old as 12, this must be the right answer.
ANSWER: b

On to Lesson #3!

Lesson #3: The Back Door Play

Transforming Algebra Problems into Arithmetic Problems

The trick I am about to teach you is my favorite SAT trick of all time. It's the trick that, more so than any other, makes you feel like you are getting away with something. I call it the Back Door, because at the key moment in the problem, instead of going through the front door with algebra, you get to go around to the back door with arithmetic. Also, this is the SAT trick that works on school tests as well. It is not so easy to write multiple-choice questions that actually force students to use algebra. ETS rarely succeeds. This is a trick worth knowing.

Note: it is going to take me a couple of pages to explain how to use the "Back Door". But once you get the hang of it, it's easy and quick. In fact, it feels like magic. Just stay with me here for a little while…

How to Recognize a "Back Door" Problem

1. Read the question carefully (as always). Do you see any algebraic variables? (Don't forget to look at the diagrams. The SAT likes to put algebra into their geometry questions. But the Back Door play takes the algebra out of them again.) If you see variables in the question, it might be a Back Door problem.

2. Next, look at the answer choices. Do at least some of them also have algebraic variables? Now, it is almost certain that the problem you are looking at is open to the Back Door play.

Here is a typical example:

In a high school club that has 50 students as members, 20 members are seniors, *n* members are juniors and 2*n* are sophomores, where *n* is an integer and $0 < n < 10$. If the remaining members are freshmen, then in terms of *n*, what percent of the members are freshmen?

a) $7n$ %
b) $(30 - 3n)$%
c) $(50 - 3n)$%
d) $(60 - 6n)$%
e) $(100 - 6n)$%

Once you know what to look for, the characteristics that identify a Back Door Problem really jump off the page. It helps that the SAT always prints algebraic variables in italics. Also, this particular example has one other clue that screams, "I am a Back Door Problem!" It's the phrase "**…in terms of…**" which is the SAT's way of telling you that the answer you are looking for is an algebraic expression, and not a number. But wouldn't you rather work with numbers? That is precisely what the Back Door play is going to do for you – let you use numbers instead of algebraic expressions.

"But I <u>like</u> algebra!" (You, again?)

Even if you are very comfortable with algebra, learn the Back Door play thoroughly before you decide. I'm pretty good at algebra myself, but the Back Door play is so simple that I end up using it all the time.

The Back Door play in 3 Easy Steps:

1. Make up numbers for each of the variables

That's it? Just make them up? ANY numbers? Well, almost any numbers. There are a couple of rules to remember about the numbers you make up:

i. <u>The numbers you make up should be easy but not the most obvious numbers.</u> If it is possible, avoid using 0, 1 and 2. And if the problem is about angles, avoid 30, 45 and 60 degrees. But what if you forget to follow this rule, and your numbers are too obvious? Usually, the method works anyway, and if it doesn't work, it tells you so. Stay tuned…

ii. <u>The numbers you make up must fit the problem</u>. What that means is that if the problem describes the variables, you have to pick numbers that match the description. And if there is an equation that relates the variables, your numbers have to work when you plug them into the equation. Sometimes, this takes TIME. Do not rush. If your numbers don't fit the problem, you won't get anywhere. Also, the harder it is to find numbers that fit, the easier the rest of the problem will be. So take your time and get this step right.

iii. If you are making up numbers to fit an equation and you find yourself completely stuck, consider making up the numbers in a different order. Usually, one order is easier than another. Or try starting over with numbers that are easier to work with.

Let's take another look at the sample problem and make up a number…

In a high school club that has 50 students as members, 20 members are seniors, *n* members are juniors and 2*n* are sophomores, where *n* is an integer and $0 < n < 10$. If the remaining members are freshmen, then in terms of *n*, what percent of the members are freshmen?

Hmm…We need to make up a number for n. Must be an integer. Between 0 and 10.
OK, I'll try n = 5.

That's it. We are done with Step 1. And notice that you don't want to spend a lot of time tormenting yourself looking for the perfect number. Just make up a number that fits what they say and then get on with it. If you come to a point where you wish you had chosen different numbers, well then you can always change your numbers then. But for now, we've made up a number and it's on to step 2.

2. Use <u>your</u> numbers to work out the answer to <u>their</u> question.

Start by re-reading the question, now that you have replaced the variables with numbers. It will seem easier. And then, to solve the problem, you will be doing arithmetic instead of algebra. The answer you get will be a number, not an algebraic expression. It won't be "in terms of" any letters. And that is OK – it's just what is supposed to happen with this method.

Let's re-read the sample problem. Now, it's <u>as if</u> it said:

In a high school club that has 50 students as members, 20 members are seniors, 5 members are juniors and 10 are sophomores. If the remaining members are freshmen, what percent of the members are freshmen?

It looks so much simpler after you replace the n with a 5 and the 2n with a 10.
And, of course, we are going to ignore the whole "in terms of n" business, and do this as an arithmetic problem.

Let's see…50 students total…20 + 5 + 10 = 35 accounted for…50 –35 = 15 so 15 of the 50 are freshman. And 15 out of 50 is 30%. So my answer is 30%.

And that's it for step 2. Do not let it bother you that your answer (30%) does not look like any of the answer choices. Step 3 will take care of that for us.

3. Go around to the Back Door!

Look at the answer choices. The reason none of them match your answer yet is that your answer was based on the decision to let n = 5. So now, you have to substitute n = 5 into each of the answer choices. Then, any choice that doesn't match your answer is wrong!

Let's see what happens. Remember, our answer is 30 and our value for n is 5…

a) 7*n* % ***7x5=35…which is not 30***

b) (30 – 3*n*)% ***30 – 3x5 = 30 –15 = 15…which is not 30***

c) (50 – 3*n*)% ***50 – 3x5 = 50 – 15 = 35…which is not 30***

d) (60 – 6*n*)% ***60 – 6x5 = 60 – 30 = 30…AHA! We have a match!***

e) (100 – 6*n*)% ***100 – 6x5 = 100 – 30 = 70…which is not 30.***

And so the answer is D.

You might be wondering why you should continue to check ALL of the answers after you have already found the one that matches your answer. The answer is simple: sometimes, you get more than one match. It doesn't happen too often to people who avoid the obvious numbers, but it does happen occasionally, so you always need to check all the answers and you need to know what to do if you get more than one match.

When you get more than one matching answer:

Go back to step 1 (where you made up numbers to fit the question)... Make up DIFFERENT numbers.

Then, repeat step 2 (where you solved their question). You will now have a new answer to their question.

Then, repeat step 3 (where you substitute your numbers into their answers), but you do NOT have to check all of the answers, only the answers that worked with your original numbers. They won't all work again.

Let's try another:

If a square's area = *x*, which of the following expressions gives the perimeter, *p* in terms of *x*?

a) $\frac{x}{4}$ **b)** $\frac{x^2}{2}$ **c)** $\frac{\sqrt{x}}{4}$ **d)** $4\sqrt{x}$ **e)** $2\sqrt{x}$

Step 1: Making up numbers that fit

Suppose we let x = 4. It's a nice, easy number, it is not 0, 1 or 2, and if the square has an area of 4, it will be easy to find the length of each side, which we will probably have to do later.

Step 2: Using our number to answer their question

Well, if the area of the square is 4, then each side of the square is 2 (because $\sqrt{4}=2$*) and then the perimeter is the sum of all four sides, so 2+2+2+2=8. And that's my answer: 8*

Step 3: Substitute our number into their choices, looking to see which one matches our answer

Let's see: my answer is 8, and I got that answer when I used x = 4. So when I try the answers…

a) $\frac{4}{4}=1$ **NO** **b)** $\frac{4^2}{2}=8$ **YES!** **c)** $\frac{\sqrt{4}}{4}=\frac{2}{4}$ **NO** **d)** $4\sqrt{4}=4\times 2=8$ **YES!** **e)** $2\sqrt{4}=2\times 2=4$ **NO**

Oh, NO! We have two that match my answer. OK, back to Step 1…

Well, if we let x=9, then each side of the square is now 3, and the perimeter 12.
So now, my answer is 12 and I used x = 9. Now I just have to recheck choices B and D…

b) $\frac{9^2}{2} = 40.5$ **NO** **d)** $4\sqrt{9} = 4 \times 3 = 12$ **YES!**

Boy! It's a good thing we checked all of the answers. If we had stopped after finding the first match, we would have had the wrong answer. Phew!

Let's do another…

If $p + r + 1 = n^2$, then which of the following is the average (arithmetic mean) of p and r?

a) $\frac{(n+1)(n-1)}{2}$ **b)** $\frac{n^2+1}{2}$ **c)** $2n^2 - 1$ **d)** $\frac{(n-1)^2}{2}$ **e** $2(1-n)^2$

This time, let's have you work through the method with me…

1. Make up numbers for p, r, and n.

p =	r =	n =

Wait! Do your numbers fit the question?

Add $p + r + 1$. Do you get n^2? If so, great. If not, you need to fix your numbers.

> Are you stuck? It might be that you are making up the numbers in the wrong order! Try picking a value for n first. Just about anything will do. I'll say $n = 6$. So $n^2 = 36$. Now, I just have to pick p and r so that $p + r + 1 = 36$. There are LOTS of easy ways.. I'll say p = 30, r = 5. See how it works? Now, don't just use my numbers – go find your own, starting with n.

(I'll just pause to let everybody catch up…hmmm…ok, that's enough)

Now, in step 2, we answer their question: we find the average of p and r, but we use the numbers we chose for p and r. For example, since my numbers were p = 30 and r = 5, when I find their average, I get (30 + 5)/2 = 17.5, which is MY answer. (You probably got a different answer because you used different numbers.)

Finally, it's step 3: I'm going to substitute n=6 into the answer choices. You do the same with your value of n. Here's what I get:

a) $\frac{(6+1)(6-1)}{2} = \frac{7\times 5}{2} = 17.5$ **YES**

b) $\frac{6^2+1}{2} = \frac{37}{2} = 18.5$ **NO**

c) $2\times 6^2 - 1 = 71$ **NO**

d) $\frac{(6-1)^2}{2} = \frac{25}{2} = 12.5$ **NO**

e) $2(1-6)^2 = 2(-5) = 50$ **NO**

Taa-daa! * Now you try a few…

* Like the National Football League, ETS frowns on trash-talking, gloating or taunting. But when you take an annoying algebra problem like this one and reduce it to a simple game of calculator arithmetic—well, it's hard not to gloat. Maybe it's even worth a ten-yard penalty.

Practice Set #2: Back Door Problems

1. If $x = \frac{y}{2}$ then $2y - \frac{x}{2} = ?$

a) $5y$ b) $4y$ c) $2y$ d) $\frac{5}{4}y$ e) $\frac{7}{4}y$

2. Lucy can wrap n candies per minute. The candies are then packed into boxes of 12. Which of the following expressions is equal to the number of boxes worth of candy that Lucy can wrap in w hours?

a) $\frac{nw}{12}$ b) $\frac{nw}{5}$ c) $5nw$ d) $12nw$ e) $72nw$

3. If $2x = 3y = 6z$, then the average of x and y =

a) $2.5z$ b) $3z$ c) $5z$ d) $5.5z$ e) $11z$

4. If m teaspoons of powdered concentrate are needed to make $3p$ ounces of lemonade, then $m+2$ teaspoons will be enough to make how many ounces of lemonade, of the same strength?

a) $3p + 2$ b) $3p + 6$ c) $\frac{3pm}{m+2}$ d) $\frac{3m}{(m+2)p}$ e) $\frac{3p(m+2)}{m}$

5. A container can hold a maximum of L gallons of water. It is initially 3/5 full. Then, after x gallons are removed, the remainder is divided equally into two smaller containers. How many gallons go into each of the smaller containers?

a) $\frac{3}{5}L - 2x$ b) $\frac{3}{10}L - \frac{x}{2}$ c) $\frac{3}{5}L - x$ d) $\frac{6}{5}L - x$ e) $\frac{6}{5}L - \frac{x}{2}$

6.If $\frac{1}{4a} = b$ and $\frac{1}{4b} = c$ then $\frac{c}{a} = ?$

a) 1 b) 4 c) 16 d) $4c^2$ e) $4/c$

Set #2 Answer Key

For each of these problems, the first thing I'll show you is the numbers that I made up for the variables. In some cases, you can use the first numbers that pop into your head. Other times, you have to think for a while. If you couldn't come up with your own numbers, look and see what numbers I used. Then, go back to the problem to see if you can finish it from there yourself. But if not, continue with the explanation given in the key below.

1. We can use y = 10 and x = 5…(There's nothing special about these—they were the first I thought of.)

With those numbers, the answer to the question is $2y - \frac{x}{2} = 2 \times 10 - \frac{5}{2} = 17.5$

Then, when we substitute y = 10 into each answer choice, the only one that matches is e) $\frac{7}{4} \times 10 = 17.5$.

ANSWER: e

2. We can use n = 10 and w = 3…(Again, nothing special about these.)

With those numbers, then Lucy can wrap 10 per minute x 60 minutes in an hour = 600 candies in an hour. So in 3 hours, she can wrap 600 x 3 = 1800 candies. And they are packed in boxes of 12, so 1800/12 = 150 boxes.

Now we put our n = 10 and w = 3 into each answer choice. The only one that matches is c) 5nw = 5x10x3 = 150.
ANSWER: c

3. OK, this one requires a little more effort to come up with numbers that fit. You might have an easier time if you make up z first. For example, if you choose z=5 then 6z=30. Now you have to choose y = 10 (so that 3y=30) and you have to choose x = 15 (so that 2x=30). So now we have our numbers: z=5, y=10 and x = 15…

With those numbers, the average of x and y is (10+15)/2=12.5

And then we substitute z=5 into the answer choices and the only match is choice a) 2.5z = 2.5 x 5 = 12.5
ANSWER: a

4. After you replace the variables with numbers, this is just a ratio problem (See page 93 for help with ratios.)
So say we use m=5 and p=6.

Now the problem reads as follows:
If 5 (remember: m = 5) teaspoons of concentrate are needed to make 18 (because 3p=3x6=18) ounces of lemonade, then 7 (that's m+2=5+2=7) teaspoons will enough to make how many ounces? This is a ratio problem:
$\frac{5}{18}=\frac{7}{x}$, cross multiply, divide, get x=25.2…yuck. But don't lose heart! Continue the process.

Now substitute m=5 and p=6 into each answer choice. Use your calculator! If you are careful, you will discover that choice e) $\frac{3p(m+2)}{m}=\frac{3\times 6(5+2)}{5}=25.2$ is the one and only match! How about that.
ANSWER: e

5. Some possible numbers are L = 50 and x = 6…

If you want to be lazy here (and don't we always?) you should pick an L that is easy to take 3/5 of. That's why I chose L = 50. So the container originally held 3/5 of 50 which is 30 gallons. Then, for x, you can pick anything less than 30. I chose 6. That leaves 24 gallons. Divided in half, it will be 12 in each tank.

Then, check each answer choice and you will find that only choice b) $\frac{3}{10}L-\frac{x}{2}=\frac{3}{10}\times 50-\frac{6}{2}=12$ is a match.

ANSWER: b

6. This one looks harder than it is. But you might need your calculator to help you find numbers that ft. For example, suppose you pick a=10. Then $b=\frac{1}{40}=.025$. And then $c=\frac{1}{4\times .025}=\frac{1}{.1}=10$. So now we have our three numbers: a=10, b=.025 and c=10…

Now answer their question: $\frac{c}{a}=\frac{10}{10}=1$. (See how easy that was? When it's harder to find numbers that fit, then the actual question is always pretty easy.)

And finally, we check the answer choices. Obviously, 'a' matches, but be careful to check all of them in case others match too. But they don't. So we're done.
ANSWER: a